Linear Equations

Key Definitions

- **Equation:** An equation is a statement that sets two expressions equal to one another.
  Examples: $x + 7 = 11$ $\quad x^2 = 9$ $\quad 7 - 3x = 2 - 3x$ $\quad 4x + 7 = x + 2 + 3x + 5$
- **Solution Set:** To solve an equation means to find all values of x that make the equation true (both sides must be equal when the solution is plugged in). We call the list of these values for x the solution set.
  Example: The equation $2x + 3 = 7$ has the solution $x = 2$. In other words, the solution set of this equation is $\{2\}$.
- **Linear Equation:** A linear equation is an equation where the highest power of x is one.
- **Rational Equation:** A rational equation is an equation that contains a rational expression.

Solving Linear Equations

- **Steps:**
  1. Eliminate any parentheses.
  2. Combine like terms on each side of the equation.
  3. Isolate x on one side of the equation by using opposite operations to move terms to the other side of the equation (i.e. if a term is being subtracted on one side, you must add that term to both sides to move it to the other side).

  Example: Solve the equation $5x - (7x - 4) - 2 = 5 - (3x + 2)$

$$5x - 7x + 4 - 2 = 5 - 3x - 2$$
$$-2x + 2 = 3 - 3x$$
$$-2x + 3x + 2 = 3 - 3x + 3x$$
$$x + 2 = 3$$
$$x + 2 - 2 = 3 - 2$$
$$x = 1$$

Solving Rational Equations

- **Steps:**
  1. State the excluded values (values that make any denominator equal to 0).
  2. Eliminate fractions by multiplying every term on both sides by the Least Common Denominator.
  3. The equation is now linear and can be solved as described in the previous section.

  Example: Solve the equation $\frac{2}{a} + \frac{3}{7} = \frac{12}{7a} - \frac{1}{3}$

$$\frac{2}{a}+\frac{3}{7}=\frac{12}{7a}-\frac{1}{3} \quad x \neq 0$$

$$21a\cdot\frac{2}{a}+21a\cdot\frac{3}{7}=21a\cdot\frac{12}{7a}-21a\cdot\frac{1}{3}$$

$$21(2)+3a(3)=3(12)-7a(1)$$

$$42+9a=36-7a$$

$$42+9a+7a=36-7a+7a$$

$$42+16a=36$$

$$42-42+16a=36-42$$

$$16a=-6$$

$$\frac{1}{16}(16a)=\frac{1}{16}(-6)$$

$$a=-\frac{3}{8}$$

Example: Solve the equation $\frac{2}{2x-5}=\frac{-1}{x+3}$

$$\frac{2}{2x-5}=\frac{-1}{x+3} \quad x\neq -3, x\neq \frac{5}{2}$$

$$\frac{2}{2x-5}(2x-5)(x+3)=\frac{-1}{x+3}(2x-5)(x+3)$$

$$2(x+3)=(-1)(2x-5)$$

$$2x+6=-2x+5$$

$$2x+2x+6=-2x+2x+5$$

$$4x+6=5$$

$$4x+6-6=5-6$$

$$4x=-1$$

$$x=-\frac{1}{4}$$

Applications Involving Linear Equations

Solving Word Problems

- **<u>Steps</u>:**
  - Read the problem and identify what you are being asked to find.
  - Reread the problem and make notes of quantities and important information
  - Assign a variable to the unknown quantity (if there are two unknowns, assign the variable to the smaller of the two)
  - Set up an equation
  - Solve the equation
  - Ask whether the solution makes sense. If possible, use estimation.

Example: Two friends decide to walk to the store, then back to their apartment. One-sixth of their time was spent walking to the store. On the way, they stopped at a florist shop for 5 minutes. They spent one-third of the time at the store, and met another friend who drove them back to their apartment in two minutes. How many minutes did they spend on this trip?

Step 1: What are we being asked to find?

The number of minutes spent on the trip

Step 2: Make notes

- $\frac{1}{6}$ of trip was walking to store
- 5 minutes at the florist shop
- $\frac{1}{3}$ of trip was at the store
- 2 minutes driving to apartment

Step 3: Assign a variable

Length of trip in minutes $= x$

Step 4: Set up an equation

The length of the trip is the sum of the times spent walking, at the florist shop, at the store, and driving.

Time walking + Time @ florist + Time @ store + Time driving = Total time of trip

$$\frac{1}{6}x + 5 + \frac{1}{3}x + 2 = x$$

Step 5: Solve the equation

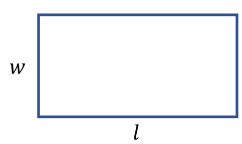
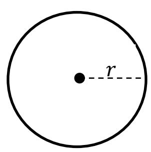
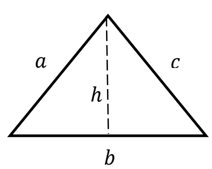
$\frac{1}{6}x + 5 + \frac{1}{3}x + 2 = x \rightarrow 5 + 2 = x - \frac{1}{6}x - \frac{1}{3}x \rightarrow 7 = x\left(1 - \frac{1}{6} - \frac{1}{3}\right) \rightarrow$
$7 = x\left(\frac{6-1-2}{6}\right) \rightarrow 7 = \frac{1}{2}x \rightarrow 2 \cdot 7 = x \rightarrow x = 14$

Step 6: Does the answer make sense?

In step 5, we found that $x = 14$. We plug this into the equation we created to check the solution.

$$\frac{1}{6}(14) + 5 + \frac{1}{3}(14) + 2 = \frac{14}{6} + 5 + \frac{14}{3} + 2 = \frac{14}{6} + \frac{30}{6} + \frac{28}{6} + \frac{12}{6} = \frac{84}{6} = 14$$

Geometric Formulas

Shape	Perimeter	Area
Rectangle 	$P = 2l + 2w$ $l = length$ $w = width$	$A = l \cdot w$
Circle 	$C = 2\pi r$ $r = radius$	$A = \pi r^2$
Triangle 	$P = a + b + c$ $a = side\ 1$ $c = side\ 2$ $b = base\ of\ triangle$ $h = height$	$A = \frac{1}{2}bh$

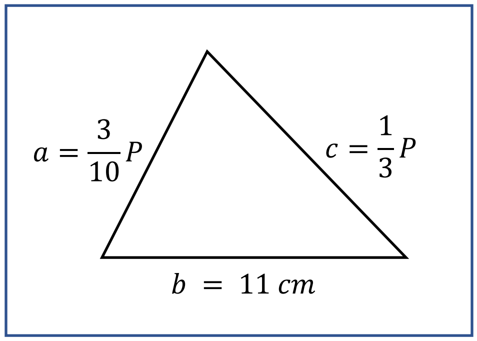
Example: Find the perimeter of a triangle if one side is 11 cm, another side is $\frac{3}{10}$ of the perimeter, and the third side is $\frac{1}{3}$ of the perimeter.

Step 1: What are we being asked to find?

The perimeter of the triangle.

Step 2: Make notes

- one side is 11 cm long
- another side is $\frac{3}{10}$ of the perimeter
- the third side is $\frac{1}{3}$ of the perimeter



Step 3: Assign a variable

P = perimeter

Step 4: Set up an equation

$$P = a + b + c = \frac{3}{10}P + 11 + \frac{1}{3}P$$

Step 5: Solve the equation

$P = \frac{3}{10}P + 11 + \frac{1}{3}P \rightarrow P - \frac{3}{10}P - \frac{1}{3}P = 11 \rightarrow P\left(1 - \frac{3}{10} - \frac{1}{3}\right) = 11 \rightarrow$
$P\left(\frac{30-9-10}{30}\right) = 11 \rightarrow \frac{11}{30}P = 11 \rightarrow \frac{\cancel{30}}{\cancel{11}} \cdot \frac{\cancel{11}}{\cancel{30}}P = \cancel{11} \cdot \frac{30}{\cancel{11}} \rightarrow P = 30$

Step 6: Does the answer make sense?

$$a = \frac{3}{10} \cdot 30 = 9 \qquad b = 11 \qquad c = \frac{1}{3} \cdot 30 = 10$$

$$P = a + b + c = 9 + 11 + 10 = 30\ ✓$$

Interest

- **<u>Principal:</u>** The principal is the total amount borrowed.
- **<u>Simple Interest:</u>** If a principal of P dollars is borrowed for a period of t years at annual interest rate r, the interest I is $I = Prt$.

  Example: A college student on summer vacation was able to make $5000 by working a full-time job every summer. He invested half the money in a mutual fund and half the money in a stock that yielded four times as much interest as the mutual fund. After a year he earned $250 in interest. What were the interest rates of the mutual fund and stock?

  Step 1: What are we being asked to find?

  The interest rates of the mutual fund and stock.

  Step 2: Make notes and assign variables.

  - $I_1 = the\ interest\ of\ the\ mutual\ fund$
  - $I_2 = 4I_1 = the\ interest\ of\ the\ stock$
  - $t = 1\ year$
  - $P_1 = \$2500\ because\ half\ the\ money\ went\ into\ the\ mutual\ fund$
  - $P_2 = \$2500\ because\ half\ the\ money\ went\ into\ a\ stock$

  Step 3: Set up equations.

  $$I_1 + I_2 = \$250 \qquad I_1 = P_1 r_1 t \qquad I_2 = P_2 r_2 t$$

Step 4: Solve the equations.

$$I_1 + I_2 = I_1 + 4I_1 = 5I_1 = 250 \to I_1 = \frac{250}{5} = 50$$
$$I_2 = 4I_1 = 4 \cdot 50 = 200$$
$$I_1 = P_1 r_1 t \to 50 = 2500(r_1)(1) \to r_1 = \frac{50}{2500} = .02 = 2\%$$
$$I_2 = P_2 r_2 t \to 200 = 2500(r_2)(1) \to r_2 = \frac{200}{2500} = .08 = 8\%$$

Step 5: Does the solution make sense?

$$I_1 = 2500(.02)(1) = 50$$
$$I_2 = 2500(.08)(1) = 200$$
$$The\ interest\ from\ the\ stock\ is\ \$200\ which\ is\ four\ times\ the\ interest\ of\ \$50\ from\ the\ mutual\ fund$$