Sampling Distributions

Key Definitions

- **Sample Distribution of the Sample Mean:** The probability distribution for all possible values of a random variable computed from a sample of size n from a population with mean μ and standard deviation σ.
- **Standard Error of the Mean**: The standard deviation of the sampling distribution is also the standard error of the mean.
- **Central Limit Theorem**: For sample sizes 30 and bigger, the sample distribution is approximately normal.
- **Sample Proportion**: The statistic that estimates the population proportion.

Mean and Standard Deviation of a Sampling Distribution

- ***Understanding the Mean and Standard Deviation of a Sampling Distribution:*** If we have a simple random sample of size n that is drawn from a population with mean μ and standard deviation σ, we can find the mean and standard deviation of a sample from that population. This is done using the information about our sample size and the information known from the population.
- ***How to Find the Mean and Standard Deviations of a Sampling Distribution:*** Finding the mean and standard deviation of a sampling distribution is very straight-forward. First, for the mean we do not have to do anything. The mean for the sampling distribution $\mu_{\bar{x}}$ is the same as the mean of the population. We show this as the following:

$$\mu_{\bar{x}} = \mu$$

To find the standard deviation of the sampling distribution, we take the standard deviation of the population, σ, and we divide it by the square root of the sample size. This will gives us the standard deviation of the sampling distribution. This is shown as the following:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- ***Example of Finding the Mean and Standard Deviation of a Sampling Distribution:***
At the end of your time with Regent University, you will take a test on your general education knowledge. Scores are approximately normally distributed with a mean score of 86 and a standard deviation of 6. What is the mean and standard deviation of the sampling distribution for a sample size of $n = 12$?
We first identify the information we already know which is the following:

$$\mu = 86 \quad and \quad \sigma = 6$$

Now, we can find the mean of the sampling distribution first:

$$\mu_{\bar{x}} = \mu$$
$$\mu_{\bar{x}} = 86$$

Next, we will find the standard deviation of the sampling distribution:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{\bar{x}} = \frac{6}{\sqrt{12}} = \frac{6}{3.46410162} \approx 1.73$$

Finding the Probability of a Sampling Distribution

- ***How to Find the Probability of a Sampling Distribution:*** If our sampling distribution is normally distributed, you can find the probability by using the standard normal distribution chart and a modified z-score formula. The modified z-score formula is the following:
$$z = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$
We use the standard normal distribution chart in the back of your book as normal.
- ***Example of Finding the Probability of a Sampling Distribution:***
At the end of your time with Regent University, you will take a test on your general education knowledge. Scores are approximately normally distributed with a mean score of 86 and a standard deviation of 6. What is the probability of a student scoring less than 90 on the test?
Since we have already found the mean and standard deviation of the sampling distribution previously, we need to plug the information into the equation.
$$z = \frac{x - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$
$$z = \frac{90 - 86}{1.73} = 2.31$$
Now that we have the z-score, we look at our standard normal distribution to find the probability.
$$P(x < 90) = 0.9896$$

Sampling Distribution of a Sample Proportion

- ***What Makes this Sampling Distribution Different:*** When we do not have certain characteristics of a sampling distribution, it is still possible to still find the sampling distribution, but we must find it using the sample proportion. This could be thought of as the number of successes over the number of trials like a binomial distribution.
- ***How to Find the Sample Proportion:*** The sample proportion is found by taking our number of individuals in the sample (or the number of successes) and dividing it by the sample size (or the number of trials). This is shown using the following formula:
$$\hat{p} = \frac{x}{n}$$
- ***Example of Finding the Sample Proportion:***
A survey was conducted where the students in a class were asked if they felt they had passed or failed the test. Out of the 35 students surveyed, 28 said they felted they had passed the test. Find the sample proportion of the individuals that felt that they passed the test.
First thing we do is identify that our $n = 35$ and our $x = 28$. From there, we plug it into our equation:

$$\hat{p} = \frac{x}{n}$$

$$\hat{p} = \frac{28}{35} = 0.8$$

Mean and Standard Deviation of a Sampling Distribution of a Sample Proportion

- ***How to Find the Mean and Standard Deviation of a Sampling Distribution of a Sample Proportion:***
  Finding the mean and standard deviation is very straight forward, and it relies on the information we know about the proportion found earlier. If we cannot find the sample proportion, we cannot find the mean and standard deviation of a sampling proportion. To find the mean of the sampling proportion, you only need to know the sampling proportion as the mean is the sampling proportion. The following represents how it shown:
  $$\mu_{\hat{p}} = p$$
  To find the standard deviation of a sampling proportion, you need to know the sampling proportion and the sample size. From here, it is very similar to how we find the standard deviation of a binomial distribution. We multiply the sampling proportion by one minus the sampling proportion. Then, we divide the multiplication by n. Finally, we take the square root of the division. The following shows us the formula to find the standard deviation of a sampling proportion:
  $$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$
- ***Example of Finding the Mean and Standard Deviation of a Sampling Distribution of a Sample Proportion:***
  A survey was conducted where the students in a class were asked if they felt they had passed or failed the test. Out of the 35 students surveyed, 28 said they felted they had passed the test. Find the mean and standard deviation of the sampling proportion of the individuals that felt that they passed the test.
  We know from our work done previously that $p = 0.8$. Now, we plug into our formulas. First, we will find the mean:
  $$\mu_{\hat{p}} = p$$
  $$\mu_{\hat{p}} = 0.8$$
  Next, we will find the standard deviation by plugging into the formula for our standard deviation of a sampling proportion:
  $$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$
  $$\sigma_{\hat{p}} = \sqrt{\frac{0.8(1-0.8)}{35}} = \sqrt{\frac{0.8(0.2)}{35}} = \sqrt{\frac{0.16}{35}} = \sqrt{0.00457143} \approx 0.07$$

Probability of a Sampling Distribution of a Sampling Proportion

- ***How to Find the Probability of a Sampling Distribution of a Sampling Proportion:*** If our sampling distribution of a sampling proportion is approximately normal (if $n\hat{p}(1-\hat{p}) \geq 10$), then we can find a probability from the sampling distribution. We do this by plugging into a modified version of the z-score formula and using the standard normal distribution table as normal. The modified version of the z-score formula is the following:

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

- ***Example of Finding the Probability of a Sampling Distribution of a Sampling Proportion:***
A survey was conducted where the students in a class were asked if they felt they had passed or failed the test. Out of the 50 students surveyed, 28 said they felted they had passed the test. The mean of the sampling distribution is $\mu_{\hat{p}} = 0.56$ and the standard deviation of the sampling distribution is $\sigma_{\hat{p}} = 0.07$. What is the probability that less than 42% have passed the test?
The first thing we will do is identify what we know:

$$n = 50, \qquad \hat{p} = 0.42, \qquad \mu_{\hat{p}} = 0.56, \qquad \sigma_{\hat{p}} = 0.07$$

The next thing we need to do is check that our sampling distribution is approximately normal.

$$n\hat{p}(1-\hat{p}) \geq 10$$
$$(50)(0.42)(1-0.42) = 12.18 \ \geq 10$$

Since our distribution is approximately normal, we can now plug into the modified z-score formula:

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

$$z = \frac{.42 - 0.56}{0.07} = \frac{-0.14}{0.07} = -2.0$$

Now that we know the z-score, we can find the probability using the standard normal distribution chart. When we use the chart, we find out that $P(z < -2.0) = 0.0228$.

Symbol Guide

Chapter Title Symbols		
Term	Symbol	Use
μ	Population Mean	To identify the population mean
σ	Population Standard Deviation	To identify the population standard deviation
$\mu_{\bar{x}}$	Mean of a Sampling Distribution	To identify the sampling distribution mean
$\sigma_{\bar{x}}$	Standard Deviation of a Sampling Distribution	To identify the sampling distribution standard deviation
$\hat{p}$	Sampling Proportion	To identify the sampling distribution's sampling proportion
$\mu_{\hat{p}}$	Mean of a Sampling Proportion	To identify the sampling proportion's mean
$\sigma_{\hat{p}}$	Standard Deviation of a Sampling Proportion	To identify the sampling proportion's standard deviation