Measures of Central Tendency

Key Definitions

- **Arithmetic Mean:** Known more commonly as the average or just mean, the arithmetic mean is the average of the quantitative data collected. You find the arithmetic mean by adding together all the data, and dividing that data by the number of data points in the set.
- **Population Arithmetic Mean:** We find the population arithmetic mean the same way we find the arithmetic mean. This is referred to simply as the population mean. We use the population mean symbol (see below) if our data set is a population. The population mean is a parameter.
- **Sample Arithmetic Mean:** We find the sample arithmetic mean the same way we find the arithmetic mean. This is referred to simply as the sample mean. We use the sample mean symbol (see below) if our data set is a sample. The sample mean is a statistic.
- **Median:** Another measure of the quantitative data is found by placing all the numbers in order from smallest to largest, and then choosing the one in the middle of the data.
- **Mode:** Another measure of data is found by observing which value appears the most in the data set. We can find the mode for either qualitative or quantitative data.
- **Outlier:** The outlier is a data value that is not close to or similar to the other data values.
- **Class Midpoint:** The sum of two consecutive lower class limits that are then divide by 2.
- **Approximate Mean of a Variable from a Frequency Distribution:** This is the average of a quantitative, continuous data set. This mean could be either a population mean or a sample mean depending on whether your data set is a population or sample data set. We find this by taking the sum of each product of the midpoint and frequency and dividing it by the sum of the frequencies.
- **Weighted Mean:** When the values of data have different levels of importance, we must account for the weight each value holds. A student's GPA is a weighted mean since each class holds a different level of weight to your grade. We find the weighted mean by taking the sum of the product of each weight and value and dividing it by the sum of the weights.

Arithmetic Mean, Median, and Mode

- *Population Mean:* The population mean is represented by the symbol, μ, in statistics. The population mean is the average we primarily use when we have a population data set. We find the population mean by adding together all of the data and then dividing it by the number of data points in the set. The formula is the following:

$$\mu = \frac{\sum x}{N}$$

- *Example of Population Mean:*
  A class of 15 had taken an exam last week. The teacher is asked for the average of the exam scores for the class. The scores are as follows:

  78, 82, 79, 78, 54, 90, 86, 91, 98, 91, 84, 89, 78, 91, 95

We plug the data into the formula above:
$$\mu = \frac{78 + 82 + 79 + 78 + 54 + 90 + 86 + 91 + 98 + 91 + 84 + 89 + 78 + 91 + 95}{15}$$
$$\mu = \frac{1264}{15} = 84.27$$

- **Sample Mean:** The sample mean is represented by the symbol, $\bar{x}$, in statistics. The sample mean is the average we primarily use when we have a sample data set. We find the sample mean by adding together all of the data and then dividing it by the number of data points in the set. The formula is the following:

$$\bar{x} = \frac{\sum x}{n}$$

- **Example of Sample Mean:**
  A class of 10 took an exam last week. The teacher is asked for the average score of the exam scores for the class. The scores are as follows:
  90, 98, 94, 75, 78, 83, 75, 72, 73, 85
  We plug the data into the formula above:
  $$\bar{x} = \frac{90 + 98 + 94 + 75 + 78 + 83 + 75 + 72 + 73 + 85}{10}$$
  $$\bar{x} = \frac{817}{10} = 82.3$$

- **Median:** The median is represented by the symbol, M, in statistics. We use the median to find the average when we have an outlier in our data set. We find the median by ordering the data set from smallest to largest. When we have an odd sample size, we either cross one out from the left and one from the right repeatedly until there is only one value left, or we divide the sample size by 2 ($\frac{n}{2}$) and round it up to find out what spot the median is in the ordered data set. When we have an even sample size, we either cross out one from the left and one from the right until we have two numbers left in the center, or we divide $\frac{n}{2}$ and $\frac{n+1}{2}$ to find out the spots where the numbers in the middle of the ordered data set are located. From there, we add the two numbers from the ordered data set together and divide it by 2 to find the median.

- **Example of Odd Sample Size Median:**
  Taking the data set from the example of the population mean, we organize the data in order from smallest to largest. When we organize the data, we get the following:
  54, 78, 78, 78, 79, 82, 84, 86, 89, 90, 91, 91, 91, 95, 98
  We cross out one from the left and then one from the right until we are left with one value in the middle. Or, we can take $\frac{N}{2}$ and round up to find out what spot the median is in. We would take $\frac{15}{2} = 7.5 \approx 8$. The median is the 8th number in the ordered list. Therefore, the median equals 86.

- **Example of Even Sample Size Median:**
  Taking the data set from the example of the sample mean, we organize the data in order from smallest to largest. When we organize the data, we get the following:
  72, 73, 75, 75, 78, 83, 85, 90, 94, 98
  We cross out one from the left and one from the right until we are left we two values in the middle of the data set, or we take $\frac{n}{2}$ and $\frac{n+1}{2}$ and round them up if they have a decimal to find out what spot the two values to find the median are. We would take $\frac{10}{2} = 5$ and $\frac{10+1}{2} = 5.5 \approx 6$. The

numbers needed to find the median are in the 5th and 6th spot. We take those the numbers 78 and 83, add them together, and divide by 2.

$$M = \frac{78 + 83}{2} = 80.5$$

- **_Mode:_** The mode does not have a symbol that represents it. We use this measure when we have qualitative data. We find the mode by counting how many times each value appears in the data set. The one that appears the most is the mode. You can have no mode (all of the values appear the same number of times), one mode (one value appears more than the rest), or multiple modes (two or more values appear an equal amount of times that are more than the rest).

- **_Example of a Single Mode:_**
  Using the data from the example of the sample mean, we can see what numbers appear the most frequently. We take our data set and mark how many time each number has appeared.

  72, 73, 75, 75, 78, 83, 85, 90, 94, 98

  The number 75 appears 2 times in the data set. Since no other numbers appears as much as this number, it is the mode.

- **_Example of Multiple Modes:_**
  Using the data from the example of the population mean, we can see what numbers appear frequently. We take our data set and mark how many times each number has appeared.

  54, 78, 78, 78, 79, 82, 84, 86, 89, 90, 91, 91, 91, 95, 98

  The numbers 78 and 91 appeared 3 times in the data set. Since no other numbers appears as much as these two numbers, they are both the mode of the data set.

*There is an easier way to compute the mean, median, and mode in Excel. If you would like to learn how to use Excel to compute the mean, median, and mode, we have an Excel spreadsheet with instructions on our website.

Approximate Mean of a Variable from a Frequency Distribution

- **_How to Find the Class Midpoint:_** To find the class midpoint, you take the two consecutive lower class limits (the first lower class limit is the one for the class you are finding the midpoint and the next lower class limit is the next class that is after the class you are finding the midpoint for), add them together, and then divide the sum by 2. The midpoint of a frequency distribution is represented by *x*.

- **_How to Find the Mean of a Frequency Distribution:_** The mean of a frequency distribution is represented by the same symbols as the sample mean and population mean (frequency distributions are either sample or population). To find the mean of a frequency distribution, you first multiply each class midpoint (*x*) by the class' frequency. After you multiply each midpoint by its frequency, you add together all the products of the midpoint and frequency. Afterwards, you divide the sum of the products of the midpoint and frequency by the sum of the frequencies. The formula is the following:

$$\mu \; or \; \bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

*The *i* in the bottom corner of the *x* and *f* is used to identify which *x* and *f* we are using. If it was the first class' midpoint and frequency, the *i* would be a 1 since it is the first class.

- **_Example of the Mean of a Frequency Distribution:_**

A fast food chain has already created the following frequency distribution of how much a sample of how much customers spend at their location:

Classes	Frequency
$1.00 - $3.99	7
$4.00 – $6.99	5
$7.00 - $9.99	5
$10.00 - $12.99	3

We want to find the mean of this frequency distribution. We first will find the midpoints of each class. For the first class' midpoint, we take its lower limit ($1.00) and the next class' lower limit ($4.00), add them together, and then divide by 2.

$$midpoint \ (x_1) = \frac{1.00 + 4.00}{2} = 2.5$$

After we find each midpoint, we multiply each midpoint by its frequency. We will use a chart to organize these steps.

Classes	Frequency	Midpoint	$x_i f_i$
$1.00 - $3.99	7	$\frac{1+4}{2} = 2.5$	$(2.5)(7) = 17.5$
$4.00 - $6.99	5	$\frac{4+7}{2} = 5.5$	$(5.5)(5) = 27.5$
$7.00 - $9.99	5	$\frac{7+10}{2} = 8.5$	$(8.5)(5) = 42.5$
$10.00 - $12.99	3	$\frac{10+13}{2} = 11.5$	$(11.5)(3) = 34.5$
Totals:	20		122

We add together all the frequencies (as seen in the chart above in the totals section) and all the products of the midpoints and frequencies (as seen in the chart above in the totals section). We plug in those totals in the formula and find the mean of the frequency distribution.

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{122}{20} = 6.1$$

Weighted Mean

- *How to Find the Weighted Mean:* We find the weighted mean by first multiplying each weight by the value it corresponds with. After we do that with each weight and value, we add together all the products. We then add together all of the weights. Afterwards, we take the sum of the products of the weights and values and divide it by the sum of the weights. This gives us the weighted mean. The formula is the following:

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$$

- *Example of the Weighted Mean:* A student has completed their first semester at Regent University and has just received their grades. The student took 5 courses. The first was a College Algebra

course that was worth 4 credits, and they earned a B in the course. The second was a English 101 course that was worth 3 credits, and they earned an A in the course. The third course was Making of a Christian Mind that was worth 3 credits, and they earned a B in the course. The fourth course was a business course that was worth 3 credits, and they earned an A in the course. The fifth course was a foreign language course that was worth 4 credits, and they earned a C in the course. Find the students GPA.

We must first identify what each grade is weighted. The weight of the grade worth: A is worth 4 points, B is worth 3 points, and a C is worth 2 points. Now we can create a table to organize the information and follow the formula.

Courses	Credit Hours (x_i)	Weight (Grade) (w_i)	$w_i x_i$
Math	4	3 (B)	$(4)(3) = 12$
English	3	4 (A)	$(3)(4) = 12$
Making of a Christian Mind	3	3 (B)	$(3)(3) = 9$
Business	3	4 (A)	$(3)(4) = 12$
Foreign Language	4	2 (C)	$(4)(2) = 8$
Totals:		16	53

We plug into the formula the totals we found from our table to get the answer.

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i} = \frac{53}{16} = 3.3125$$

Symbol Guide

Measures of Central Tendency Symbols		
Term	Symbol	Use
Sum (Add Together)	Σ	To show when you need to find the sum of certain data
Population Mean	μ	Identify the population mean
Population Size	N	Identify the population size
Sample Mean	$\bar{x}$	Identify the sample mean
Sample Size	n	Identify the sample size
Median	M	Identify the median
Frequency	f	Identify the frequency
Weighted Mean	$\bar{x}_w$	Identify the weighted mean